# HEAT TRANSFER ON PLANETARY SCALES

Robert G. Watts Department of Mechanical Engineering Tulane University

# ABSTRACT

The Earth is a very complex dynamic/thermodynamic system. Predicting things like weather patterns is notoriously difficult, especially on small local scales. In fact the system of equations that predict weather behave chaotically, as has been shown by Lorenz<sup>1</sup>. But what if we are interested in some large-scale average quantities such as globally averaged surface air temperature?

# INTRODUCTION

In a local and transient sense, the thermal, physical and dynamical properties near the surface of Earth constitute the weather. I will be concerned in this brief paper not with local daily events that make up the weather, but with suitably longterm averages of seasonal or annual averages of properties. These averages are what we refer to as climate. While weather predictions exhibit chaotic oscillations, they are bounded, and when suitably averaged, the predictions are useful.

General Circulation Models are very complex models that incorporate models of the troposphere and the lower stratosphere, the ocean, land ice and snow, and often sea ice. Clouds are currently crudely parameterized, as is sea ice. Most of the important physical phenomena are included, but sometimes crudely. Models of the global temperature increase that is expected following a doubling of atmospheric carbon dioxide (after the system reaches equilibrium) typically agree within a factor of two or three. What is more important than globally averaged temperature are the regional changes of such things as temperature and soil moisture. The models in some cases differ in sign about changes in soil moisture, which is obviously very important to agriculture.

If theories are nets cast to capture the essence of the physical world, our net here is perhaps too fine. Trying to understand climatic change by using huge computer models is a bit like trying to drink from a fire-hose. Because of this a variety of simpler approximate models have been developed. These simpler models serve several purposes. They give us a broad idea of the cause and effect of long-term climate change, and they often give very valuable insight into how to better design and understand experiments with more complex models. I will first present an analysis of a global energy balance model to show what it can tell us about the temperature rise associated with a doubling of atmospheric carbon dioxide. Next, I show how the huge heat capacity of the global ocean can affect climate transients and, indeed can affect the detection of a carbon dioxide/climate signal in the atmosphere.

## NOMENCLATURE

- A + BT: Infrared flux leaving Earth/atmosphere system
- c: Specific heat in the ocean
- D: Depth of ocean mixed layer
- F: Infrared flux leaving top of atmosphere
- h: Average cloud height
- K<sub>V</sub>: Effective diffusivity of global ocean
- Q: S/4
- N: Cloud fraction
- S: Solar constant = 1360 w/m<sup>2</sup>
- t: Time
- T: Earth surface temperature, or ocean temperature
- Te: Effective Earth/atmosphere radiating temperature
- $T_{\rm P}$ : Temperature of water sinking at poles
- w: Average upwelling speed in global ocean
- x<sub>s</sub>: Area of Earth covered by ice and snow
- y<sub>C</sub>: Atmospheric CO<sub>2</sub> content
- z : Vertical coordinate
- y<sub>W</sub>: Atmospheric water vapor content
- $\alpha$ : Albedo (reflectivity) of the Earth/atmosphere
- ρ: Density of ocean water

## A Zero-dimensional Earth

First consider the simplest possible thermodynamic model of Earth. The value of the solar constant at the distance from the Sun to Earth is approximately 1360 w/m<sup>2</sup>. Because of the rotation and geometry of Earth the average amount at the top of the atmosphere is one-fourth of this. A fraction  $\alpha$  (which climate modelers call the albedo) is reflected away by clouds and the surface. F is the infrared flux leaving the top of the atmosphere. At steady state

$$(1-\alpha)S/4 = F \tag{1}$$

The albedo is a function of several things, the most important of which are the fraction of Earth covered by clouds, N, the height of the clouds, h, and the area covered by ice and snow,  $x_s$ . Thus the change in the albedo due to a change  $dy_c$  in the CO<sub>2</sub> content is

$$d\alpha = \frac{d\alpha}{dy_{c}}dy_{c} + \frac{\partial\alpha}{\partial N}\frac{dN}{dT}dT + \frac{\partial\alpha}{\partial h}\frac{dh}{dT}dT + \frac{\partial\alpha}{\partial x_{s}}\frac{dx_{s}}{dT}dT$$
(2)

The change in the infrared flux is a function mainly of the  $CO_2$  content, the water vapor content and cloud fraction and height. Thus,

$$dF = \frac{dF}{dy_C} dy_C + \frac{\partial F}{\partial T} dT + \frac{\partial F}{\partial y_W} \frac{dy_W}{dT} dT + \frac{\partial F}{\partial N} \frac{dN}{dT} dT + \frac{\partial F}{\partial h} \frac{dh}{dT} dT$$
(3)

After a perturbation in the infrared flux, dF, and once equilibrium is reached,

$$-(S/4)d\alpha - dF = 0 \tag{4}$$

From these expressions we can see that the change in temperature when the  $CO_2$  content is raised by  $dy_C$  is

The terms in the numerator represent the rate at which the net radiative flux at the top of the troposphere changes with  $CO_2$  content, all other variables remaining constant. Its value has been determined by a number of investigators using radiative-convective atmospheric values to be  $-4.2 \text{ w/m}^2$  and is generally felt to be accurate to within 25% at least.

The first term in the denominator is the rate at which the infrared flux changes with temperature, all other variables remaining constant. Its value may be estimated by supposing that the Earth/atmosphere system radiates as a blackbody at some equivalent temperature  $T_e$ , so that  $F = \sigma T_e^4$ . The appropriate temperature to be used is that seen by a viewer in space, which is approximately 265K. Thus,  $\partial F/\partial T = 3.8 \text{ w/m}^2$  °C. With no feedbacks at all doubling CO<sub>2</sub> would lead to an increase in the average temperature of the atmosphere of 4.2/3.8 = 1.1 °C.

The largest of the feedback terms in the denominator is likely the water vapor term. Atmospheric radiation studies imply that this term is approximately -2 w/m<sup>2</sup> °C with an uncertainty of perhaps 0.5w/m<sup>2</sup> °C.

Cloud feedback is the most controversial and least understood feedback. I has been variously estimated as zero to both positive and negative. I give it here a value of zero with an uncertainty of  $0.3 \text{ w/m}^2 \,^{\circ}\text{C}$ . Ice/snow albedo feedback is  $0.3 \text{ w/m}^2 \,^{\circ}\text{C}$  with an uncertainty of  $0.1 \text{ w/m}^2 \,^{\circ}\text{C}$ . Taken altogether we find that the increase in average Earth surface temperature for a doubling of CO<sub>2</sub> in the atmosphere is between 1.6 and  $4.7 \,^{\circ}\text{C}$ . It is worth noting that the above is taken from a paper published 23 years ago (Watts, ref.2). The current estimate from many GCM studies and reported by the Intergovernmental Panel on Climate Change (IPCC) <sup>3</sup> is 1.5 to 4.5  $\,^{\circ}\text{C}$ .

So we expect an increase in Earth's temperature if and when the  $CO_2$  concentration doubles. But it won't happen immediately because the heat capacity of the system, particularly the ocean, is so large. It has been only fairly recently that coupled ocean-atmosphere GCMs have been able to run in the transient state following gradually increasing atmospheric loading of  $CO_2$ . The results of transients are given in the IPCC reports, but they are not the results of GCM runs. Rather, they are the results of a slightly more complex energy balance model called an upwelling-diffusion model. It involves the same kind of radiation model as before but connected to a one-dimensional (depth only) ocean.

$$dT = \frac{-\left(\frac{\partial F}{\partial y_{c}} + \frac{S}{4}\frac{\partial \alpha}{\partial y_{c}}\right)dy_{c}}{\frac{\partial F}{\partial T} + \frac{\partial F}{\partial y_{W}}\frac{dy_{W}}{dT} + \frac{\partial F}{\partial N}\frac{dN}{dT} + \frac{\partial F}{\partial h}\frac{dh}{dT} + \frac{S}{4}\left(\frac{\partial \alpha}{\partial N}\frac{dN}{dT} + \frac{\partial \alpha}{\partial h}\frac{dh}{dT} + \frac{\partial \alpha}{\partial x_{s}}\frac{dx_{s}}{dT}\right)}$$
(5)

## A one-dimensional Earth: Transients

There has been much discussion in recent years of the influence of the ocean's thermohaline circulation in delaying the onset of greenhouse induced climate a change. The thermohaline circulation is essentially the vertical overturning of the ocean. More recently, attention has been given to how change in the strength of the thermohaline circulation can itself affect the climate. A one-dimensional ocean model provides a kind of "toy" with which to at least qualitatively understand the situation. The ocean is modeled as a mixed layer sitting atop a deep ocean. As in the real ocean, cold water at high latitudes sinks locally into the deep ocean and upwells more or less in the remainder of the ocean. The governing equation for the ocean interior is

$$\frac{\partial T}{\partial t} + w \frac{\partial T}{\partial z} = K_V \frac{\partial^2 T}{\partial z^2}$$
(6)

At the bottom of the constant depth ocean the vertical advective and diffusive terms must equal:

$$w(T - T_P) = K_V \frac{\partial T}{\partial z}$$
(7)

At z=0 (the bottom of the mixed later, where the temperature is assumed to be uniform, but not constant of course) the rate of change of thermal energy in the mixed layer is equal to the radiant energy received from the Sun minus that radiated away plus the net energy received by diffusion and advection from the deep water.

$$\frac{1}{\rho c} \left[ aQ - (A + BT) \right] - w \left( T_p - T \right) - K_V \frac{\partial T}{\partial z} = D \frac{\partial T}{\partial t} \quad (8)$$

The values of the various constants have been thoroughly discussed in the literature (see Watts, ref. 4 and Watts and Morantine, ref. 5) and are listed in the Nomenclature section.

The equations are not difficult to solve for example when the value of A is decreased to represent an increased greenhouse effect. Similarly, if the value of the upwelling speed w is perturbed, a regular perturbation problem emerges.

The globally averaged surface temperature has been reported by several groups of scientists, and of course we know that the values reported indicate a warming of between 0.4 and 0.6 °C over the past century. A puzzling aspect of these reported temperatures is that they all report a period from roughly 1040 until 1980 when there was a decided dip an the temperatures – an unexplained cooling period. Of course the climate signal is expected to be noisy and to vary on its own on many time scales. But this only begs the question. Why?

Two studies that were published in the oceanography literature may provide a possible answer. Roemmich and Wunsch<sup>6</sup> reported data taken on a transect of the North Atlantic ocean in 1957-59 with another taken in 1981. Levitus<sup>7</sup> used several million disposable bathythermograph measurements to compare the temperature and salinity structure of the North Atlantic ocean for the two pentads 1955-59 and 1970-74. Both found that the ocean at intermediate depths (between about 500m and 2500m) had warmed by an average of 0.1°C. If we assume that most of the mid-latitude North Atlantic ocean (only about 6% of the total ocean area) warmed by this amount, the necessary energy input is  $1.7 \times 10^{22}$  joules<sup>8</sup>. Taken over a period of 15 years the required rate of transfer of heat is 36 million megawatts. Averaged over the entire surface of the Earth this amounts to 0.072 wm<sup>-2</sup>. During this period the increase in surface heat flux due to greenhouse gases was about

0.125 wm<sup>-2</sup>. Thus if a similar warming of the intermediate depth ocean occurred in just 10% of the world ocean the expected loss of heat from the ocean surface would be sufficient to completely counter the greenhouse effect.

What does our model tell us? We used the model described above to see what would happen if the thermohaline circulation (the upwelling speed, w) were decreased by 10%. The figure shows the resulting temperature variation of the surface temperature superimposed on the globally averaged temperature reported by researchers at the University of East Anglia (which is essentially the same as that reported by a number of other groups). When the upwelling speed is reduced more heat diffuses into the deep ocean, warming the water at intermediate depths and cooling the surface. In the long run, the surface temperature of the surface returns to the value (in our model) dictated by radiation balance at the top, but this takes several decades. Meanwhile the greenhouse may be hiding in the deep ocean.

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